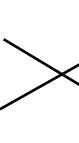


Constructing functions

Function problems

 → "8"

 → cat
 → squirrel

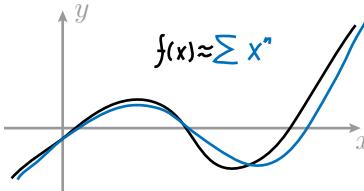
 → 

Hey,
ChatGPT! → Hey,
User!

$$x \rightarrow \boxed{f(x)} \rightarrow y$$

Taylor series

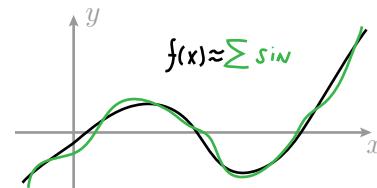
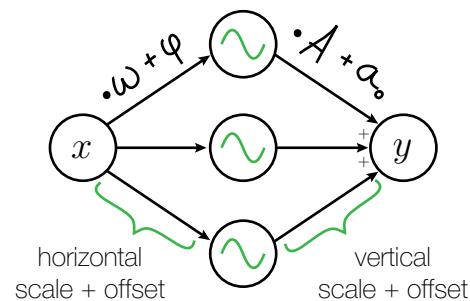
$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots$$



$$= + + \dots$$

Fourier series

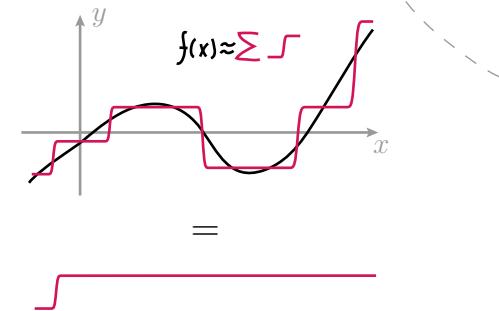
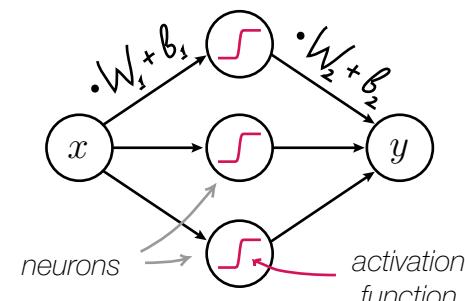
$$f(x) = \sum A \sin(\omega x + \varphi) + a_0$$



$$= + + \dots$$

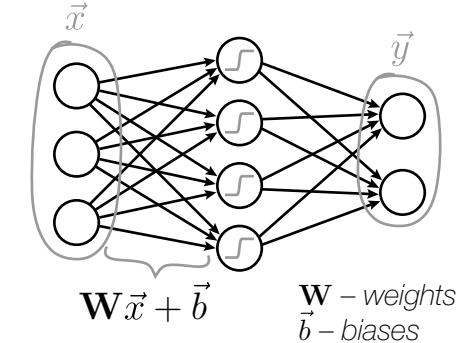
Single-layer neural network

$$f(x) = \sum W_2 \sigma(W_1 x + b_1) + b_2$$



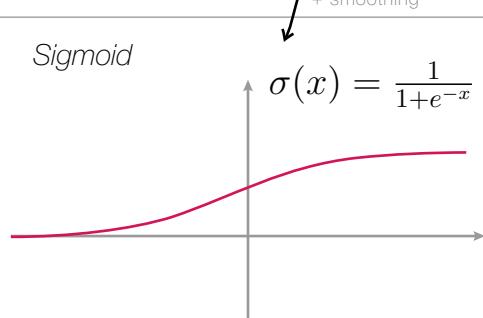
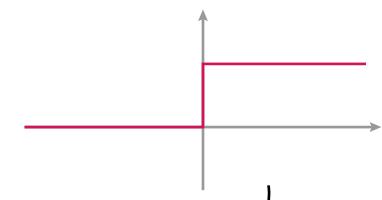
$$= + + \dots$$

Multiple dimensions

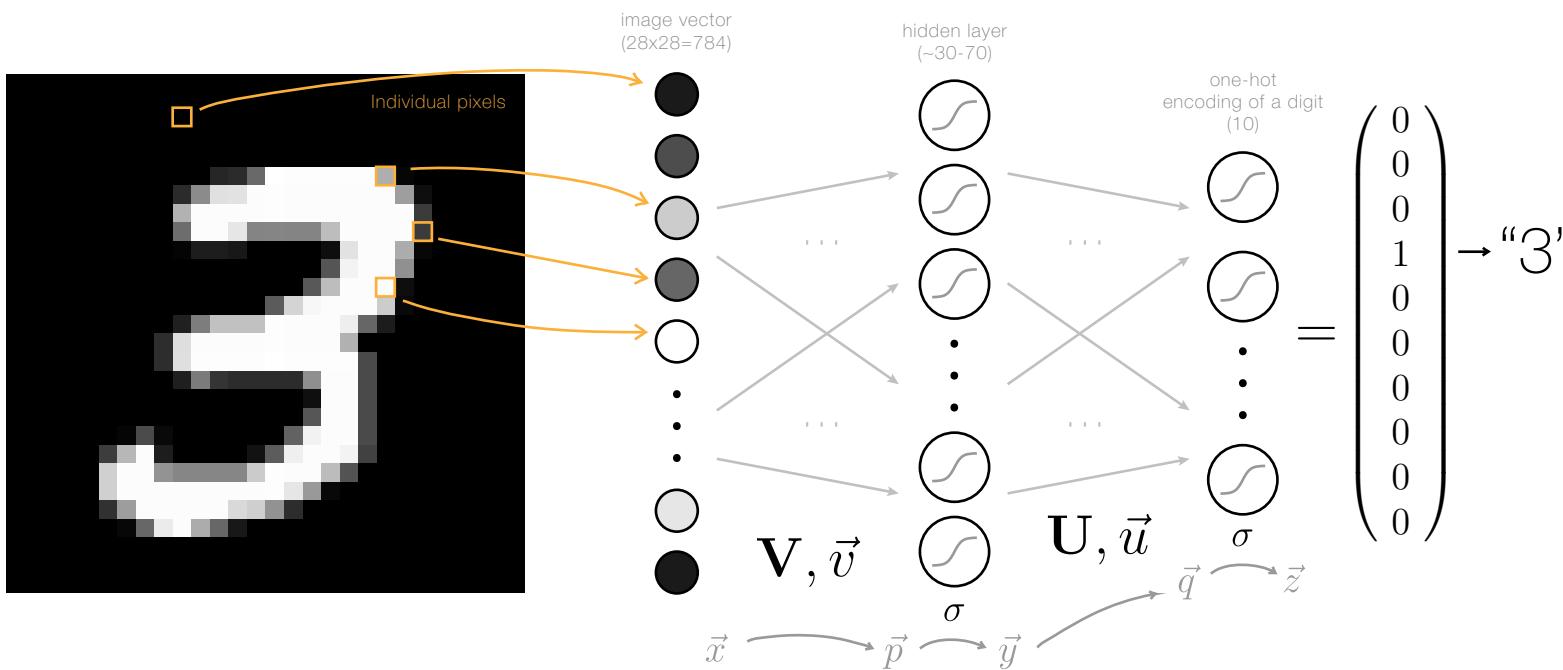


[1] Universal approximation theorem
(Cybenko)

Unit step function



Classification neural network



Forward Pass

$$\begin{aligned}
 \vec{p} &= \mathbf{V}\vec{x} + \vec{v} \\
 \vec{y} &= \sigma(\vec{p}) \\
 \vec{q} &= \mathbf{U}\vec{y} + \vec{u} \\
 \vec{z} &= \sigma(\vec{q})
 \end{aligned}$$

element-wise

\vec{z} = encoding of a digit

Digit itself ✗

$$8 \rightarrow f(x) \rightarrow 8$$

Edge case

$$\begin{aligned}
 7 &\rightarrow f(x) \rightarrow 7? \\
 4 &\rightarrow f(x) \rightarrow 4
 \end{aligned}$$

✗

One-hot encoding ✓

$$8 \rightarrow f(x) \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{matrix} \text{"0"} \\ \text{"1"} \\ \vdots \\ \text{"7"} \\ \text{"8"} \\ \text{"9"} \end{matrix}$$

Edge case

$$\begin{aligned}
 7 &\rightarrow f(x) \rightarrow \langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rangle = \begin{pmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{pmatrix} \\
 &\quad \begin{matrix} 1? \\ 7? \end{matrix} \quad \begin{matrix} \text{"1" (50%)} \\ \text{"7" (50%)} \end{matrix}
 \end{aligned}$$

✓